
Elasticity of factor substitution and the rise in labor's share of income during the Great Depression

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2009/11

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Elasticity of factor substitution and the rise in labor's share of income during the Great Depression*

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August 28, 2009

Abstract

The sudden rise in labor's share of income during the U.S. Great Depression of 1929-1933 is examined. To explain this phenomenon, the deflation-based model of the Great Depression of Bordo et al. (2000) [Bordo, M.D.; Erceg, C.J.; Evans, C.L. "Money, Sticky Wages, and the Great Depression." *American Economic Review* 90:5, 1447-63.] is extended to the case of a Constant Elasticity of factor Substitution (CES) production function. It is shown that considering the low elasticity of factor substitution allows the model to explain the rise in labor's share of income, improves the model's predictions on other macroeconomic variables, and renders the issue of productivity during the Great Depression less puzzling.

Keywords: Great Depression; Labor's share of income; CES production function; Deflation.

JEL codes: E32; E51; O47

*I thank Slim Bridji and Tristan-Pierre Maury for comments and suggestions. Any errors that remain are my own.

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1 Introduction

A striking feature of the U.S. Great Depression is the sudden rise in labor's share of income (LS hereafter), which rose by 10 percentage points in only four years between 1929 and 1933¹. This feature was measured and studied even at the time, by Simon Kuznets in his reports on national accounts for the NBER – see Kuznets (1937, IV pp. 23–27) and Kuznets (1941, chapter 6 pp. 215–256). However, the rise in LS seems to have attracted relatively little attention in the recent literature on the Great Depression².

Bordo et al. (2000), Cole et al. (2005), Chari et al. (2007), Harrison and Weder (2006), and Weder (2006) have recently developed Dynamic Stochastic and General Equilibrium (DSGE) models consistent with the major facts of the Great Depression: the sharp fall in output, employment, consumption, and investment. Unfortunately, these models are useless for understanding the rise in LS, because they share the assumption that it was constant during the Great Depression. This literature also includes the contributions of Christiano et al. (2003), Cole and Ohanian (2004), and Ebell and Ritschl (2008), who study the behavior of labor unions during the interwar period³. These contributions are certainly useful for understanding the links between the labor market and the Great Depression, but they do not study the implications of the behavior of unions on LS in the economy. In this paper, I aim to contribute to this literature by proposing a DSGE model that is consistent with the observed rise in LS during the Great Depression of 1929-1933.

Understanding fluctuations in LS has long been a major issue in macroeconomics and several mechanisms to explain the phenomenon have been proposed in the literature. These mechanisms

¹See section 2 for details of the data and of fluctuations in labor's share of income during the Great Depression.

²An exception is the recent contribution of Reicher (2008) who studies the fluctuations in LS in the context of a matching model for the labor market.

³Cole and Ohanian (2004) study the consequences of the NIRA reforms of 1933-1935 on the persistence of the Great Depression. Ebell and Ritschl (2008) focus on the bargaining process in the labor market during the 1920s up to the crisis of 1929. Christiano et al. (2003) estimate shocks to the bargaining power of unions in a DSGE model for the interwar period.

are based on either market imperfections⁴ or production technology⁵. This article contributes to this literature on fluctuations in LS, which is devoted mainly to postwar fluctuations, by assessing the relevance of these mechanisms for the interwar period. To explain the rise in LS during the Great Depression, I explore the consequences of a low (that is, below unity) elasticity of factor substitution in the economy.

The consequences of the elasticity of factor substitution on the fluctuations in LS are usually assessed by considering a Constant Elasticity of factor Substitution (CES) production function. The stake is the choice of the model into which to incorporate this production function. I incorporate this production function into the deflation-based model of the Great Depression developed by Bordo et al. (2000). In this model, the great contraction of output between 1929 and 1933 results from negative and large monetary shocks on the economy through a sticky wage channel. The choice of model is motivated by the fact that the rise in LS is concentrated within the same period (1929-1933) and by the wide consensus on the importance of monetary shocks and on the relevance of the sticky wage channel⁶. Naturally, it would be interesting to link the issue of the rise in LS with the reforms

⁴The first explanations of the countercyclical behavior of LS were based on the insurance features of the labor contract; see Boldrin and Horvath (1995) and Gomme and Greenwood (1995). There is also an extensive literature that is based on the contribution of union market power to the understanding of the structural evolution of the European labor markets; see Blanchard (2007).

⁵Recent explanations of the countercyclical behavior of LS are based on modifications of the production technology. Young (2004) and Ríos-Rull and Santaaulàlia-Llopis (2008) introduce biased technological progress shocks that drive the fluctuations in LS. Zeng (2007) proposes to abandon the Cobb-Douglas production function in favor of a Constant Elasticity of factors Substitution (CES) production function. Guo and Lansing (2009) also present results that support using a CES production function, but in the context of a model with indeterminacy. Finally, Hansen and Prescott (2005) explain the fluctuations in LS by the presence of a binding constraint on capacity in the production sector.

⁶Bordo et al. (2000) follow the popular view of Eichengreen and Sachs (1985) and Bernanke and Carey (1996), who explain the Great Depression by appeal to the links between the real wage, the deflation, and the monetary regime. The purpose of Bordo et al. (2000) is to quantify the ability of the sticky wage channel to explain the Great Depression. Chari et al. (2007) demonstrate the equivalence between the sticky wage channel and the labor wedge and show, by means of business cycle accounting methodology, that this wedge contributes to the Great Depression. For alternative points of view, see Cole et al. (2005), who conclude that productivity shocks dominate monetary shocks,

of goods and labor markets, which have been studied in detail, notably by Cole and Ohanian (2004). However, these reforms (as the *NIRA*) take place after 1933, that is, after the initial rise in LS that I try to explain in this article.

To assess the specific consequences of the elasticity of factor substitution, the model is kept as close as possible to Bordo et al. (2000) except for the production function. The model is simulated for various degrees of elasticity of factor substitution, including the Bordo et al. (2000) specification of a unit elasticity as a particular case. I show that decreasing the elasticity of factor substitution to reasonable values (about 0.30) allows the model to explain the rise in LS and improves its predictions for other macroeconomic variables: output, consumption, investment, and real wage. The only exception is the labor input variable, for which the fit is slightly worse. These results have implications for the computation of the Total Factor Productivity (TFP). With a unit elasticity of factor substitution, the empirical TFP exhibits a large fall during the Great Depression, whereas the theoretical TFP is constant in the deflation-based model. However, with lower elasticity of factor substitution, the empirical TFP contraction becomes moderate and less inconsistent with the deflation-based explanation of the Great Depression.

The remainder of the article is organized as follows. Section 2 documents the stylized facts and provides initial thoughts on explaining the rise in LS on the basis of the degree of factor substitution. Section 3 describes the model and specifies the production function. Section 4 presents the calibration procedure and the model's predictions for the Great Depression. Section 5 concludes.

2 Stylized facts

This section presents the key stylized facts and shows why they support an explanation of the rise in LS on the basis of the degree of factor substitution. To measure LS, I use the data of Kuznets and Harrison and Weder (2006), who emphasize the role of sunspot shocks.

(1941), who provides the annual decomposition of national income by type of income between 1919 and 1938 (see the Appendix A for a detailed description of the data and their sources). Figure 1 plots LS. During the 1920s, LS is roughly constant, with small fluctuations around the mean value of 75%. After 1929, LS rises sharply to 84% in 1932-1934, then moderately declines after 1934 but remains above 80% at the end of the decade. The great contraction of the U.S. economy between 1929 and 1933 is then characterized by a huge growth in LS. Gomme and Ruppert (2007) compute LS using the NIPA annual database, which starts in 1929, and obtain a similar increase of LS during the 1930^s; see Figure 1 of Gomme and Ruppert (2007). Table 1 reports the values of LS using the Kuznets and NIPA databases between 1929 and 1933 and the log deviation with respect to the value of 1929. While the levels of the two series differ⁷, they show the same growth over the period from 1929 to 1933 (the two log deviations with respect to the 1929 value are near 13% in 1933).

Table 1 also reports the data for the ratio of capital to labor. A salient feature of the data is the similarity of the growth rates of LS and of the capital-labor ratio during the Great Depression. Both series exhibits a similar high and rapid increase for the period from 1929 to 1933. This observation motivates the assumption of a CES production function that links the LS to the capital-labor ratio. To make this point explicit, I consider the following specification of the production function:

$$Y_t = F(K_t, L_t) = A \left[\phi K_t^{(\sigma-1)/\sigma} + (1 - \phi) L_t^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \quad (1)$$

where Y_t is the output, K_t is the stock of physical capital, L_t is the labor input, $A > 0$ is the efficiency parameter, $0 < \phi < 1$ is the distribution parameter, and $\sigma > 0$ is the elasticity of substitution between capital and labor. When $\sigma = 1$, the function $F(K_t, L_t)$ corresponds to the Cobb-Douglas production technology with an elasticity of factor substitution equal to unity. Capital and labor are perfect substitutes for $\sigma \rightarrow \infty$ and strictly complementary for $\sigma \rightarrow 0$. Without market imperfections, profit maximization by firms implies that the wage is equal to the marginal productivity of labor. Using

⁷The difference between the levels of share can be explained by the fact that the two databases do not provide the same decomposition of the national income by type of income. Interestingly, Young (2004) and Ríos-Rull and Santaefulàlia-Llopis (2008) show that different definitions of LS, while differing on their average, have very similar business cycle properties for post-war data.

Equation (1) to compute the marginal productivity of labor $F_2(K_t, L_t)$, the expression for LS s_t is:

$$s_t = \frac{F_2(K_t, L_t) L_t}{Y_t} = \left[\left(\frac{\phi}{1 - \phi} \right) k_t^{\frac{\sigma - 1}{\sigma}} + 1 \right]^{-1} \quad (2)$$

LS, s_t , is a function of the capital intensity of labor $k_t = K_t/L_t$ (that is, the ratio of capital to labor) and of the structural parameters: ϕ and σ .

In this setup, fluctuations in LS result from fluctuations in the capital-labor ratio and the parameter σ determines the sign and the sensitivity of the relation between these two variables. With variables in log deviation form, Equation (2) implies:

$$\widehat{s}_t = \left(\frac{\sigma - 1}{\sigma} \right) \widehat{k}_t \quad (3)$$

where $\widehat{x}_t = \log(x_t/x)$ is the log deviation of x_t with respect to the steady-state value x for $x = \widetilde{s}, k$ and $\widetilde{s}_t = (1 - s_t)/s_t$ is the ratio of the physical capital share of income to LS. The data reported in Table 1 can be used to set the appropriate value of σ to account for the fluctuations between 1929 and 1933. The log deviation of the capital-labor ratio, $\widehat{k}_{1933} = 32.19\%$, leads to the log deviation of LS, $\widehat{s}_{1933} = 13.00\%$, for $\sigma = 0.33$. This value of σ means that there is low substitutability between physical capital and labor. It is worthy of note that this value is below, but close to, the range of conventional estimates of degree of substitution of factors. Chirinko (2008) provides a survey of these estimates and suggests that $[0.40 - 0.60]$ is a possible range for σ .

To conclude this section, the simultaneous rise of LS and of the capital-labor ratio during the Great Depression can be founded on a CES production function, with low elasticity of factor substitution. However, this observation is not sufficient to demonstrate the relevance of this mechanism for explaining the rise in LS, because quantities of output and input are been taken as given here. To demonstrate the relevance of this mechanism, it is necessary to show that the CES production function is consistent with an endogenous determination of output and input quantities. To achieve this aim, I incorporate the CES production function into the deflation-based model of the Great Depression of Bordo et al. (2000) and study the consequences of a lower substitutability between

physical capital and labor than that supposed by the authors⁸.

3 The sticky wage model with CES production function

This section summarizes the model of Bordo et al. (2000) and extends it to the case of a CES production function. The program of the representative household is to maximize the following expected and discounted utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\mu \log(C_t) + (1 - \mu) \log\left(\frac{M_t}{P_t}\right) \right] \quad (4)$$

where β is the discount factor, μ is the utility function parameter, C_t is the real consumption, M_t is the end-of-period nominal cash balances, and P_t is the price level. The budget constraint of households is:

$$B_t = B_{t-1} + (R_{t-1}B_{t-1} + W_tL_t + J_tK_t + \Pi_t + X_t) - (P_tC_t + P_tI_t + M_t - M_{t-1}) \quad (5)$$

where B_t is nominal bond holdings, R_t is the nominal interest rate on bonds, W_t is the nominal wage rate, L_t is the total number of hours worked, J_t is the rental price of capital in nominal terms, K_t is the capital supplied to firms, Π_t is nominal firm profits, X_t is lump-sum cash transfers from the government, and I_t is gross real investment. Households accumulate capital that they rent to firms according to:

$$K_{t+1} = (1 - \delta) K_t + I_t \quad (6)$$

where K_t is the physical capital stock and δ the rate of depreciation. I consider the benchmark case of Bordo et al. (2000) without labor adjustment costs in the production sector. The program of the representative firm is to choose the quantities of physical capital and labor to maximize per-period profits:

$$P_tF(K_t, L_t) - W_tL_t - J_tK_t \quad (7)$$

⁸The elasticity of factor substitution is equal to one in Bordo et al. (2000).

where the prices of factors of production are taken as given. The production $F(K_t, L_t)$ is defined by equation (1). Nominal rigidities in the labor market are introduced with overlapping wage contracts, where the contract wage X_t as of time t depends on the expected values for future nominal wages and employment rates. More precisely, the wage contract is

$$\begin{aligned} \log(X_t) = & \frac{1}{4} \log(W_t) + \gamma(L_t - \bar{L}) + \text{E}_t \left\{ \frac{1}{4} \log(W_{t+1}) + \gamma(L_{t+1} - \bar{L}) + \frac{1}{4} \log(W_{t+2}) \right. \\ & \left. + \gamma(L_{t+2} - \bar{L}) + \frac{1}{4} \log(W_{t+3}) + \frac{1}{4} (L_{t+3} - \bar{L}) \right\} \end{aligned} \quad (8)$$

and the aggregate nominal wage in the economy is the geometric average wage of all cohorts in the economy:

$$W_t = X_t^{\phi_0} X_{t-1}^{\phi_1} X_{t-2}^{\phi_2} X_{t-3}^{\phi_3} \quad (9)$$

Fluctuations in this economy are driven by exogenous variations in the growth rate of the stock of money: $g_t = \log(M_t) - \log(M_{t-1})$. The growth rate g_t is assumed to follow an autoregressive process with one lag:

$$g_{t+1} = g_0 + \rho g_t + \varepsilon_{t+1} \quad (10)$$

where g_0 is a constant, ρ is the persistence of process, and $\varepsilon_t \sim N(0, \sigma_\varepsilon)$ is the innovation of the process.

4 The model's predictions

4.1 Calibration

The model is calibrated and simulated to compare the outcomes of endogenous variables to historical monetary shocks during the Great Depression for different values of σ . The calibration procedure is based on Bordo et al. (2000). The following parameters are set to conventional values. The discount factor is $\beta = 0.99$ and the depreciation rate of physical capital is $\delta = 0.025$. The steady-state values of employment is set to one-third, $L = 1/3$, and of output to one, $Y = 1$. The utility function

parameter μ does not influence the model dynamics. Monetary shocks are constructed with equation (10), where the parameters $g_0 = 0.0035$ and $\rho = 0.485$ are estimated using the database of Balke and Gordon (1986) for the stock of money. The AR(1) is estimated with the M1 monetary aggregate for the period 1922(2):1928(4) and monetary shocks ε_t deduced for the period 1929(1)-1933(4).

For the two parameters A and ϕ of the production function, I apply the normalization procedure of Klump et al. (2007) and Klump and Saam (2008). These parameters are recalibrated when σ changes to ensure the same steady-state values for the capital to output ratio ($K/Y = 7.12$) and LS ($s = 0.75$) in the model. It is recommended that this normalization procedure be used when comparing the model's simulations with different values for the elasticity of factor substitution.

As in Bordo et al. (2000), the parameter γ is allowed to vary with the specification of the production process. Indeed, Bordo et al. (2000) consider two values for γ , according to the presence of labor adjustment costs. The parameter γ is adjusted here when the elasticity of substitution between capital and labor varies. The parameter γ is set to minimize the sample Mean Square Error (MSE) between the observed and simulated series⁹ of output¹⁰:

$$\hat{\omega}_y = \min_{\hat{\gamma}} \frac{1}{4} \sum_{t=1930}^{1933} (\hat{y}_{t,1929}^{\text{data}} - \hat{y}_{t,1929}^{\text{model}}(\sigma))^2 \quad (11)$$

where $\hat{y}_{t,1929} = \log(y_t/y_{1929})$ is the log deviation of the output y_t as of time t with respect to the value for 1929. Log deviations are denoted by $\hat{y}_{t,1929}^{\text{data}}$ for the data and $\hat{y}_{t,1929}^{\text{model}}(\sigma)$ for the model, given the value of σ . To assess the consequences of the degree of factor substitution, the range of values for σ is $[0.2, 1.2]$. For each value of σ , the parameter γ is set to its optimal value $\hat{\gamma}$ according to (11)

⁹Since historical data are annual, the quarterly data simulated by the model are converted to annual frequency by taking the average.

¹⁰Bordo et al. (2000) choose γ to minimize the sample MSE of real wage and not of output. I also tested this criterion without modifications of the results reported in the next section. By choosing to minimize output's MSE, the contribution of the mechanism based on factor substitution is assessed in a context in which the sticky wage channel's ability to reproduce the contraction of output is optimal (including in the benchmark case of $\sigma = 1$).

and the parameters ϕ and A are recalibrated following the normalization procedure described above.

4.2 The sticky wage channel with low elasticity of factor substitution

This section presents the consequences of the parameter σ for the model's predictions. Figure 2 gives a first summary of the results for the two key variables of interest: output and LS. This Figure depicts the sample MSEs of output, $\hat{\omega}_y$, and of LS¹¹, $\hat{\omega}_s$, for the values of $\sigma \in [0.2, 1.2]$.

The benchmark case is $\sigma = 1$. In this case, the model is equivalent to that of Bordo et al. (2000), who consider a Cobb-Douglas production function. The cases of $\sigma > 1$, which correspond to high degrees of substitution between capital and labor, are unattractive, because the two sample MSEs of output and LS go up¹². These results confirm the conclusion of Section 2, such that low elasticity of factor substitution is better than high. The cases of $\sigma < 1$, which correspond to low degrees of substitution between capital and labor, improve the predictions of the model. The two sample MSEs of output and LS are lower for $\sigma \in [0.2, 1[$ than in the benchmark case (that is, $\sigma = 1$), but for all that, the lowest elasticity of factor substitution is not the best choice. Indeed, the relation between the sample MSEs and the elasticity of factor substitution is U-Shaped. In the remainder of this section, three calibrations are compared.

- Calibration 1 is the benchmark case and corresponds to the case of a constant LS, i.e., $\sigma = 1$.
- Calibration 2 minimizes the sample MSE of output for $\sigma = 0.381$.
- Calibration 3 minimizes the sample MSE of LS for $\sigma = 0.296$.

These values for calibrations 2 and 3 are very close to the value of σ computed in Section 2

¹¹The formula for computing the sample MSE of LS is the same as for output defined in Equation (11).

¹²More precisely, for $\sigma > 1$: the simulated LS decreases during the Great Depression, which is contrary to the empirical facts, and the simulated contraction of output becomes larger than the contraction of output observed in the data.

using the historical data for LS and for the capital to labor ratio¹³. Table 2 gives the values for the parameters $\{\gamma, \phi, A\}$. As explained in section 4.1, the values of parameters A and ϕ are adjusted to ensure steady-state constraints and the value of γ is set to maximize the model's ability to replicate the contraction in output of the period 1929-1933. With this calibration procedure, the values of parameters γ and σ are related positively; in Table 2, γ goes from 0.045 to 0.007 while σ goes from 1 to 0.296. Lower γ means higher sensitivity of the labor input to changes in nominal wages; see Equation (8) of the wage contract. When factors of production are less substitutable in the economy (which means that σ diminishes), labor input has to be more sensitive to nominal wages (which means that γ diminishes) to maximize the model's fit for output.

The Impulse Response Functions (IRFs) help to understand the relation between σ and γ . Figure 3 plots the IRFs of output and LS to a positive monetary shock in three cases. The first case corresponds to the benchmark case $\{\sigma = 1.000, \gamma = 0.045\}$. In the second case, σ is set to the value that minimizes the sample MSE of LS, but the value of γ is unchanged $\{\sigma = 0.296, \gamma = 0.045\}$. In the third case, the value of γ is adjusted to minimize the sample MSE of output when the degree of factor substitution is low $\{\sigma = 0.296, \gamma = 0.007\}$. Decreasing the value of σ to less than one makes the LS volatile and countercyclical, but dampens the effects of the shock on the output. The amplitude of the output's response to the monetary shock is weaker at all horizons when the elasticity of factor substitution is low (see the lines with diamonds and with circles in Figure 3). Given the inertia of the physical capital stock in the economy, the amplitude of the labor input's response to the shock is smaller when factors are less substitutable. Consequently, if the low substitution of factors amplifies the response of LS to monetary shocks, it also dampens the response of output. To compensate for this last effect, the sticky wage channel is magnified by considering lower values for γ (see the lines with diamonds and with squares in Figure 3).

¹³As indicated in Section 2, these values are below, but not too far below, the range of plausible values for σ proposed by Chirinko (2008) ($0.40 < \sigma < 0.60$). They are also close to the value suggested by Zeng (2007) ($\sigma = 0.50$) to account for the business cycle fluctuations in LS in the post-war U.S. economy.

Figure 4 compares the predictions that the model makes on the basis of the historical data for selected macroeconomic variables: output, LS, labor input, investment, consumption, and the real wage. The model's predictions are for the benchmark case $\sigma = 1$ and for the optimal values of $\sigma = \{0.381, 0.296\}$, in order to replicate either the output contraction or the rise in LS. For $\sigma = 1$, the model recovers the satisfying properties of Bordo et al. (2000). The deflation induced by monetary shocks explains relatively well the contraction of output between 1929 and 1933 and the dynamics of other key variables except for LS. Indeed, by construction, the model with $\sigma = 1$ predicts a constant LS during the Great Depression (see the line with triangles). On the other hand, with low elasticity of factor substitution, the model explains almost two-thirds of the rise in LS for $\sigma = 0.381$ (see the line with diamonds) and almost all the rise for $\sigma = 0.296$ (see the line with squares). The fit of the model is also improved for other variables. Table 3 reports the sample MSEs. The contribution of low factor substitution does not concern only LS. The sample MSEs of output, consumption, investment, and the real wage diminish when σ decreases from one to below unity. The only exception is the labor input, which sample MSE slightly augments. To conclude this section, the low elasticity of factor substitution makes the deflation-based model of the Great Depression consistent with the rise in the labor's share and improves the model's performances for other key macroeconomic variables.

4.3 The puzzle of productivity during the Great Depression with low elasticity of factor substitution

The puzzle of productivity during the Great Depression refers to the large and unexplained fall in the Total Factor Productivity (TFP) during the period from 1929 to 1933¹⁴. This puzzle is based on measures of the TFP that use the Cobb-Douglas production function, as in Cole and Ohanian (1999), Cole et al. (2005), Weder (2006), and Chari et al. (2007). However, the previous results

¹⁴As proposed by, for example, Cole et al. (2005) in their paper entitled "Deflation and the International Great Depression: A Productivity Puzzle". Ohanian (2001) provides a detailed discussion on the fall of the TFP during the Great Depression.

suggest that this specification induces an elasticity of factor substitution in the economy that is too high. For the deflation-based model of Bordo et al. (2000), I show in Section 4.2 that using a CES production function with an elasticity of factor substitution below one is better for explaining the Great Depression than using the Cobb-Douglas production function. It is worth mentioning that this choice of production function has strong implications for the computation of the TFP. For example, Klump et al. (2007) report significant differences between estimates of the TFP according to the elasticity of factor substitution for the postwar U.S. economy. Consequently, I propose to assess the implications of low factor substitutability on the measure of the TFP for the Great Depression. To this end, the production function (1) is extended to the case of a variable TFP, denoted by A_t , which is computed as follows:

$$\log(A_t) = \log(Y_t) - \left(\frac{\sigma}{\sigma-1}\right) \log\left(\phi K_t^{(\sigma-1)/\sigma} + (1-\phi) L_t^{(\sigma-1)/\sigma}\right) \quad (12)$$

The empirical TFP depends on the historical series (for output, Y_t , physical capital, K_t , and labor input, L_t) and on the structural parameters (ϕ and σ). In the benchmark case $\sigma = 1$, the remaining parameter ϕ is usually set to the average value of the physical capital's share of income. For $\sigma < 1$, the empirical TFP can be very different from the benchmark case, even if the historical series for output and inputs are the same.

Figure 5 compares the empirical TFP for the parameters values of Table 2, $\{\sigma = 1, \phi = 0.25\}$ and $\{\sigma = 0.296, \phi = 0.998\}$, with the theoretical TFP. For the deflation-based model presented in Section 3, without technological shocks, the theoretical TFP is constant during the Great Depression whatever the elasticity of factor substitution (see the line with squares). For $\{\sigma = 1, \phi = 0.25\}$, the empirical TFP falls rapidly, about 22% between 1929 and 1933, and then returns slowly to the 1929 value at the end of the 1930s (see the line with circles). The picture is radically different for $\{\sigma = 0.296, \phi = 0.998\}$. The TFP still falls at the beginning of the 1930s, but not with the same amplitude (the fall between 1929 and 1933 is about 6% of the 1929 value instead of 22% in the other case), and then exceeds the 1929 value after 1934 (see the line with diamonds). With low, but reasonable, elasticity of factor substitution the empirical TFP does not fall to any great extent

during the Great Depression¹⁵. This result is critical for assessing the relevance of the deflation-based model of the Great Depression.

For $\{\sigma = 1, \phi = 0.25\}$, the theoretical TFP in the deflation-based model is very far from the empirical TFP. With Cole et al. (2005), one can wonder in this case how the deflation-based model would behave if such technological shocks were added to the monetary shocks¹⁶. For $\{\sigma = 0.296, \phi = 0.998\}$, the conclusion is radically different. The constant theoretical TFP is not too far from the empirical TFP. Given the evidence reported in Section 4.2 in support of low elasticity of factor substitution, the behavior of the TFP between 1929 and 1933 does not disqualify the deflation-based model of the Great Depression.

5 Concluding remarks

This paper was motivated by the evidence of a sudden rise in the LS during the U.S. Great Depression. To explain this phenomenon, I extended the deflation-based model of Bordo et al. (2000) to the case of a non-unit elasticity of factor substitution. A low, but reasonable, value of elasticity of factor substitution (near 0.30) allows this model to account for the rise in the LS and improves its performance for other key macroeconomic variables. These results reinforce the credibility of the deflation-based explanation of the Great Depression and suggest that the low elasticity of factor substitution is a suitable mechanism for explaining fluctuation's in LS.

¹⁵Interestingly, Chari et al. (2007) report a similar shift in their measure of the efficiency wedge when they consider a variable utilization rate of capital; see Figure 9 p.815 in Chari et al. (2007).

¹⁶The deflation-based model is expected to strongly overestimate the fall in output during the Great Depression if the exogenous variations of the TFP depicted in Figure 5 for $\{\sigma = 1, \phi = 0.25\}$ are added to the monetary shocks considered in Section 4.2.

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A Data

- **Labor share's of income** is measured using series of labor income Y_t^L and of physical capital income Y_t^K . Series of mixed incomes are not necessary, if they are assumed to combine labor and physical capital incomes in the same proportion as in the overall economy – see Gomme and Ruppert (1997). Labor's share of income s_t is computed as follows:

$$s_t = \frac{Y_t^L}{Y_t^L + Y_t^K}$$

with two databases for Y_t^L and Y_t^K . The reference database is Kuznets (1941, Table 22(c) p. 218): $Y_t^L = (\text{Wage and salaries}) + (\text{other payments to employees})$ and $Y_t^K = (\text{Corporate Rent}) + (\text{Net Dividends}) + (\text{Covered Interest}) + (\text{Net Saving})$. The second database is the Section 2 - Personal Income and Outlays provided by the NIPA: $Y_t^L = (\text{Compensation of employees Other B203RC1})$ and $Y_t^K = (\text{Rental income of persons A048RC1}) + (\text{Corporate profits A051RC1}) + (\text{Net interest and miscellaneous payments W255RC1})$.

- **Real wage data** is constructed as the ratio of the monthly series of nominal wages provided by Hanes (1996) to the quarterly series of GNP's deflator provided by Balke and Gordon (1986). Series are taken on average to have the same periodicity.
- **Money stock** is the quarterly monetary aggregate $M1$ of Balke and Gordon (1986).
- **Other per capital and real macroeconomic aggregates (output, labor input, physical capital, consumption, and investment)** are taken from the annual database of Chari et al. (2007), which is based on the database of Kendrick (1961). See Chari et al. (2007) for a detailed description of the data construction.

B Legend of figures

Figure 1. The historical labor's share of income 1919-1938 in %. Source: Kuznets (1941).

Figure 2. The sample Mean Square Errors (MSEs) of the output and the labor's share of income for elasticity of factor substitution σ between $[0.20; 1.20]$. The sample MSE of output is the solid line (left-scale) and the sample MSE of labor's share of income is the dotted line (right-scale). For each value of σ , parameter values γ , ϕ , and A are adjusted according to the calibration procedure described in Section 4.1 (parameter values are reported in Table 2).

Figure 3. Impulse Response Functions (IRFs) of output and labor's share of income to a positive monetary shock. The line with circles refers to the benchmark case $\{\sigma = 1.000, \gamma = 0.045\}$. The line with diamonds refers to the model with low elasticity of substitution, but unchanged sensibility of labor input to nominal wages $\{\sigma = 0.296, \gamma = 0.045\}$. The line with squares refers to the model with low elasticity of factor substitution and adjusted sensibility of labor input to wages $\{\sigma = 0.296, \gamma = 0.007\}$.

Figure 4. Historical data and the model's predictions for output, labor's share of income, labor input, investment, consumption, and real wage. The line with circles are for the data and the line with triangles for the model in the benchmark case $\sigma = 1$, the line with diamonds are for the model with $\sigma = 0.381$ that minimizes the sample MSE of output, and the line with squares for the model with $\sigma = 0.296$ that minimizes the sample MSE of labor's share of income.

Figure 5. Log deviation of the Total Factor Productivity (TFP) with respect to the value for 1929. The line with circles refers to the empirical TFP using the value of $\sigma \rightarrow 1.000$ and the solid lines with diamonds refers to the empirical TFP using the value of $\sigma = 0.296$ that minimizes the sample MSE of labor's share of income. The solid lines with squares refers to the model's prediction for TFP whatever the value of σ .

C Tables

Years	Labor share Kuznets (1941) data		Labor share NIPA data		Capital to labor ratio: $k_t = K_t/L_t$	
	s_t	\hat{s}_t	s_t	\hat{s}_t	$k_t/1000$	\hat{k}_t
1929	74.04	—	67.81	—	119.83	—
1930	75.69	2.21	69.73	2.79	135.25	12.11
1931	79.87	7.59	73.54	8.11	155.74	26.21
1932	84.53	13.25	76.35	11.86	176.12	38.51
1933	84.31	13.00	77.85	13.81	165.33	32.19

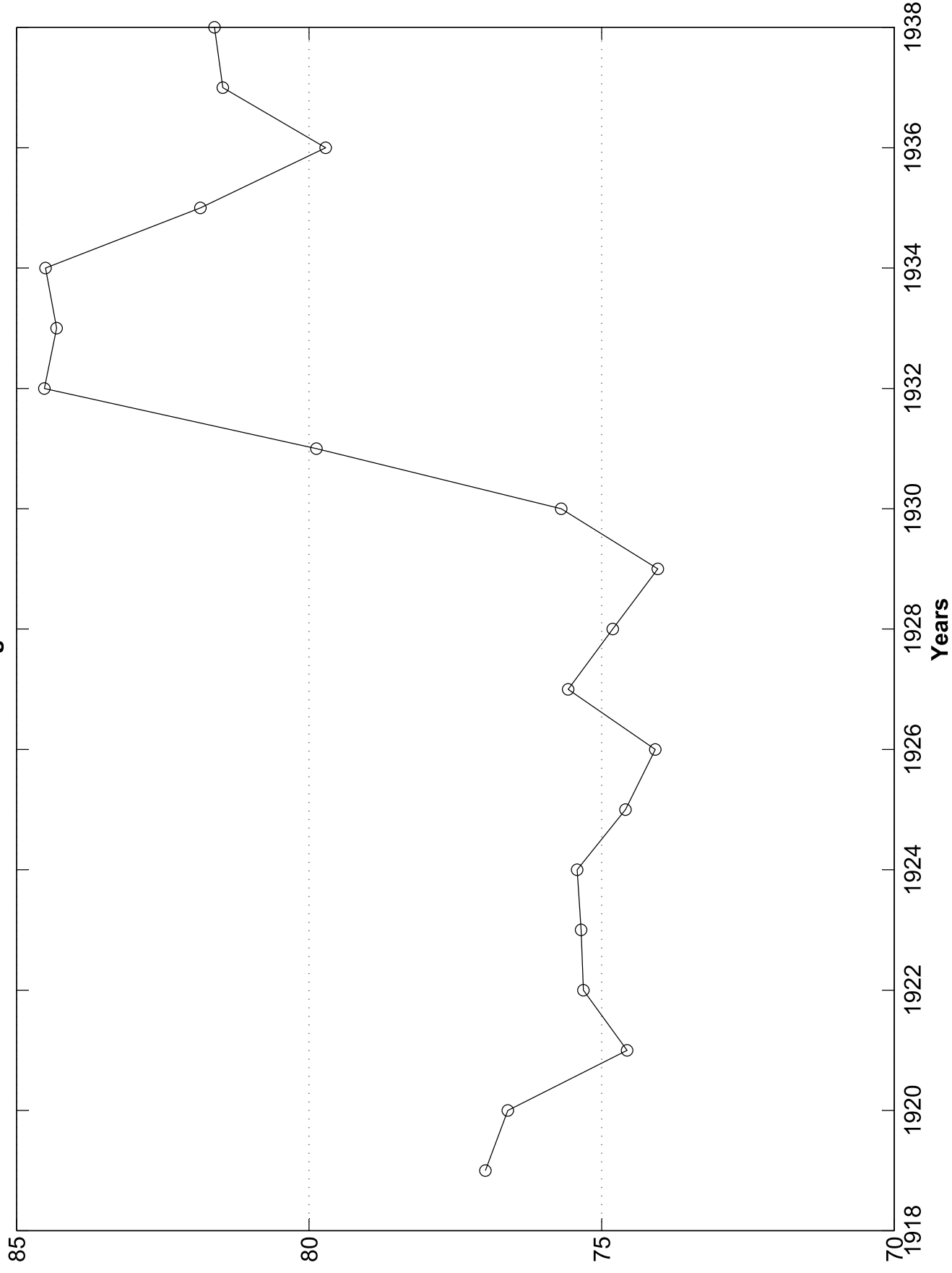
Table 1. Historical data and log deviation with respect to the value of 1929 (in percent) for the labor's share measured with the Kuznets (1941) and NIPA databases and for the capital to labor ratio.

Calibration	Parameters			
	σ	γ	ϕ	A
1. Benchmark case (Cobb-Douglas)	1.000	0.045	0.250	1.395
2. To minimize the sample MSE of output	0.381	0.013	0.979	0.325
3. To minimize the sample MSE of labor's share	0.296	0.007	0.998	0.251

Table 2. Values for σ and the dependant parameters: γ , ϕ , and A . See the text for the other parameters.

Variable	Elasticity of substitution		
	$\sigma = 0.296$	$\sigma = 0.381$	$\sigma = 1.000$
Output	0.0020	0.0010	0.0128
Labor share	0.0003	0.0052	0.0406
Labor input	0.0235	0.0191	0.0189
Investment	0.0685	0.0649	0.1794
Consumption	0.0008	0.0012	0.0068
Real wage	0.0133	0.0073	0.0385
Table 3. Sample Mean Square Errors (MSE) of selected macroeconomic variables according to the elasticity of factor substitution.			

Figure 1



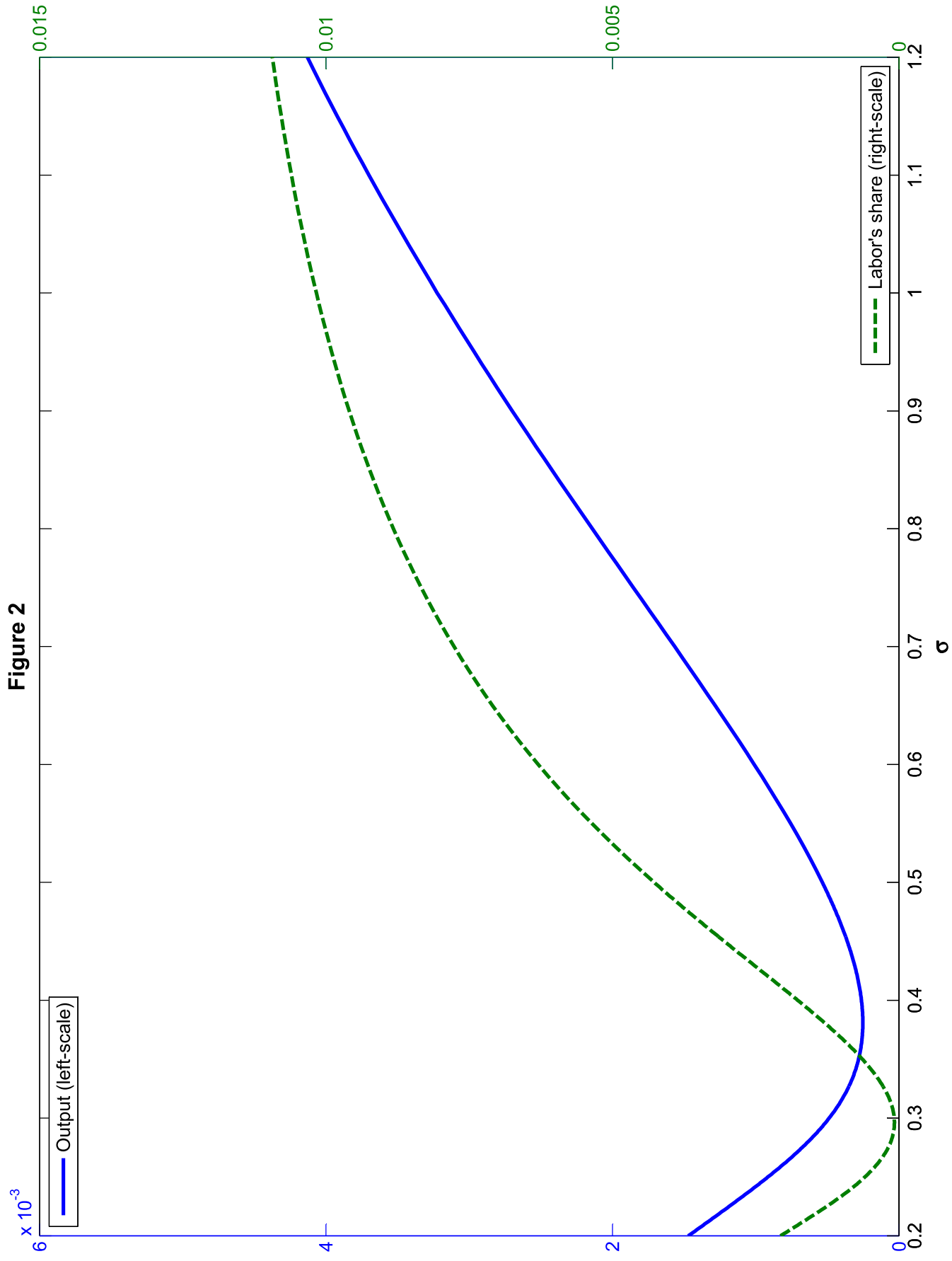


Figure 3

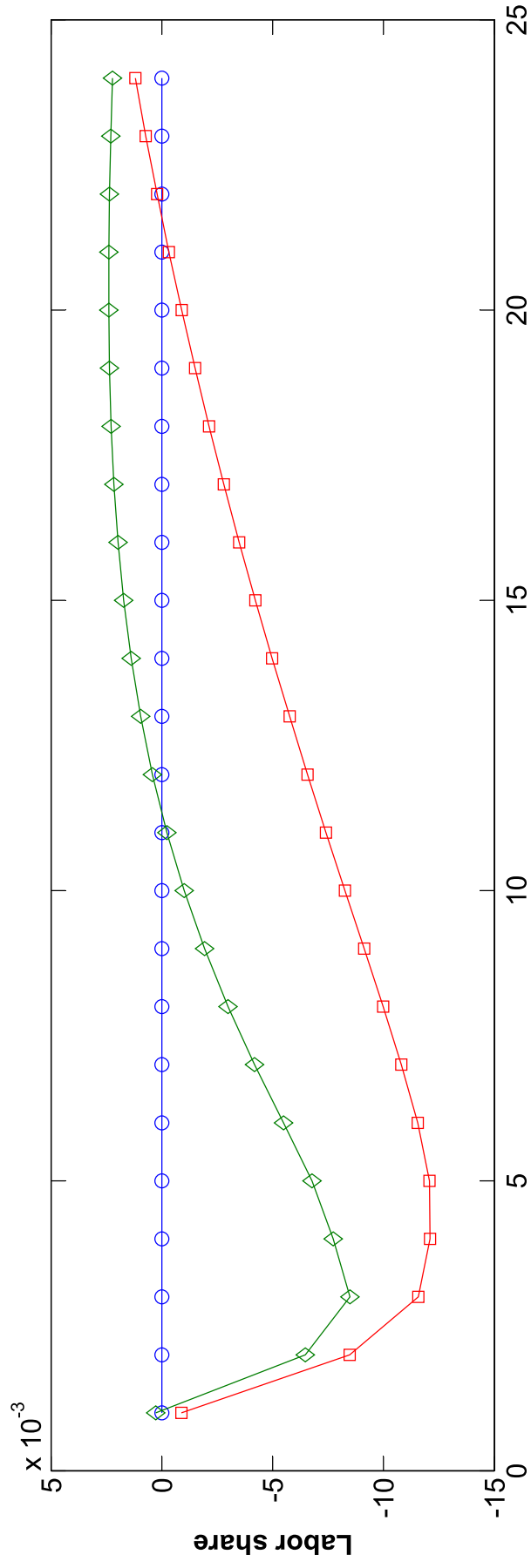
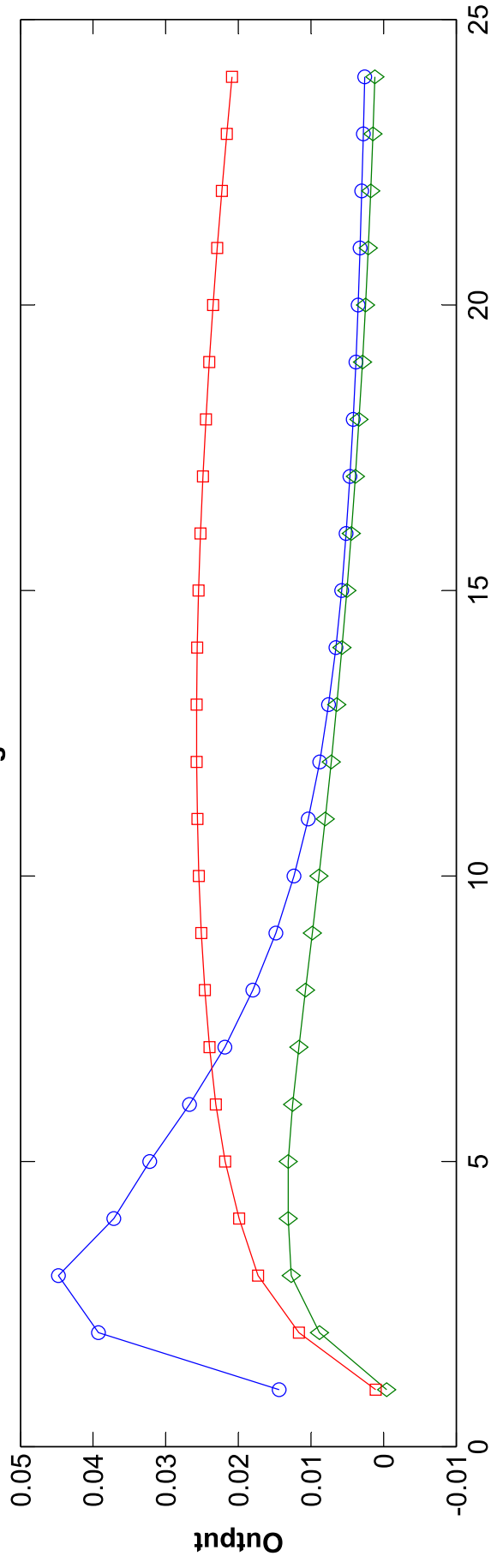


Figure 4

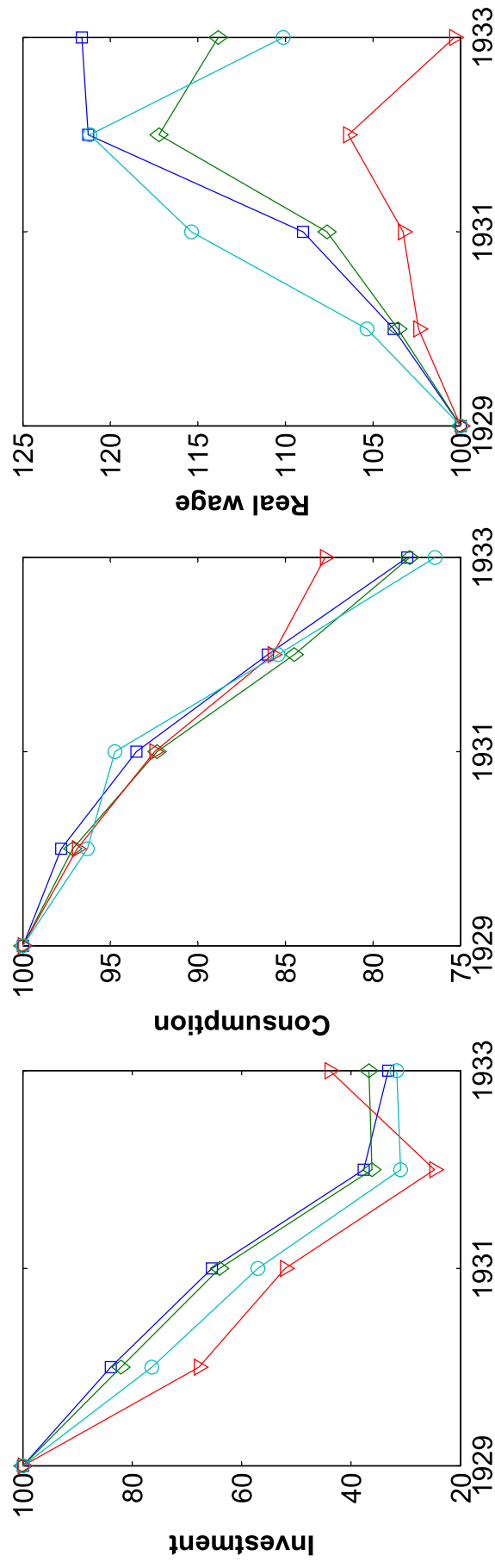
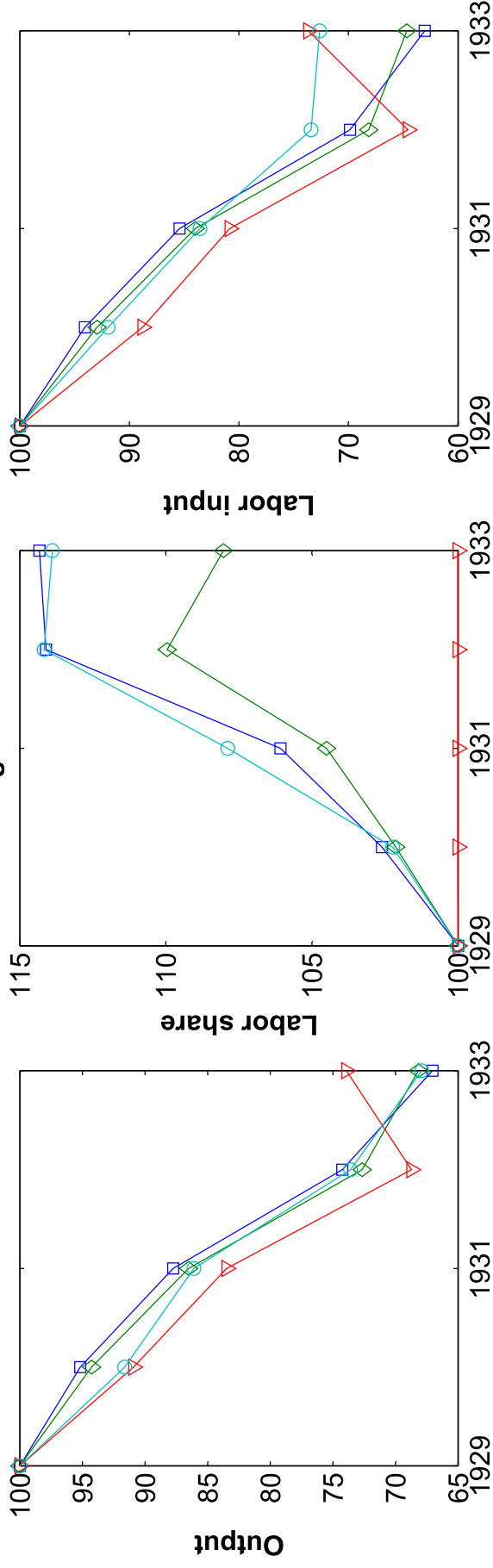


Figure 5

